Realistic Throughput of Cellular Multi-Hop Relay Networks With Spatial Reuse

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***Abstract*—Multi-hop relaying can increase the data capacity of cellular systems by reducing path loss, mitigating shadowing, and enabling spatial reuse. However, the analysis of capacity in relay networks remains an open problem. In this paper we calculate throughput regions for some specific cellular multi-hop relay network topologies allowing for non-uniform traffic distribution. There is a trade-off between serving subscriber stations near and far from the base station.**

Key words: cellular networks, multi-hop relaying, WiMAX, HiperMAN, dimensioning.

1. Introduction

Multi-hop relaying [1], [2], [3], is a key technique expected to improve data transmission rates and area coverage in next generation wireless data systems. With relaying, data destined for a mobile station can be relayed via multiple radio hops rather than being transmitted directly from the distant base station. Although relaying requires the use of additional radio resources (frequency channels or time slots) relaying can significantly reduce the path loss by shortening the propagation path and routing around obstacles. Reduced path loss translates to increased transmission rates and spectral efficiency. In addition, spatial reuse (SR) is enabled, which allows multiple transmissions to take place simultaneously throughout a cell. Multi-hop relaying is an option in 802.16-2004 (Mesh mode) [4] and is currently studied in 802.16e work (mobile multi-hop relaying - MMR) [5], [6]. Similarly it is a key part of High Performance Radio Metropolitan Area Network (HiperMAN) standard [7], [8]. The combination of spatial reuse time division multiple access (STDMA) [9], [10], [11], [12], in which transmission time slots are reused in geographically distant areas of a cell, with multi-hop relaying provides an even greater benefit. However, the number of relays introduced and the dimensions of coverage areas of relays must be carefully chosen for the specific propagation environment and system parameters, otherwise poorer efficiency may result. In addition, the specific geometry of the system (Manhattan versus hexagonal cell tessellations) and the spatial reuse scheduling mechanism have a great impact on the network throughput.

Much work exists on the theoretical capacity of ad-hoc relay networks ([13], [14], [15], [16], [17], [18] are a few examples), generally under a number of simplifying assumptions. Bounds on the capacity of ad-hoc networks have been derived for classes of networks with random topologies, where the bounds hold with high probability as the network size gets large (asymptotically). In a network containing numerous nodes, each pair of which may communicate, the network capacity can be described by capacity regions. *Capacity* is defined as the maximum rate at which data transmission is achievable between two nodes. With N_n nodes in a network, there are $N_n(N_n-1)$ rates between node pairs. The set of all such rate combinations is called the *capacity region* and has dimensionality $N_n(N_n-1)$. The shape of capacity regions for wireless networks depends on numerous factors: data transmission schedule, propagation environment, etc., and are very difficult to derive analytically. Capacity bounds from the cited work give useful bounds on performance for classes of random and arbitrary ad-hoc networks, but do not give specific design rules or actual throughput for a realistic cellular system. The goal of our work is to develop design rules that could be incorporated into system design software. In order to do so we find specific performance of networks by calculating signal to interference and noise ratios (SINRs) for numerous system topologies and cluster sizes (using parameters and techniques used in 802.16 and HiperMAN), finding each link's throughput using adaptive modulation and coding (AMC), and calculating the resulting network throughput. Overhead in the physical and medium access control layers is included to determine usable *throughput*. In this paper, we define a *throughput region*, similar to the capacity region, as the set of all usable throughputs between nodes or groups of nodes. Calculations for hexagonal and Manhattan tree topologies have been done for two to four relay hops. Throughput regions for a three-hop hexagonal cellular layout are presented in this paper as an example.

In Section 2 we present the system model, in Section 3 we demonstrate results using one particular example, and we conclude the paper in Section 4.

2. System Model

2.1. Topology

A two-dimensional multi-hop relaying cellular system has macrocells of circumscribed radius, r, each with a base station (BS) at its centre. Each macrocell is then partitioned into numerous microcells, each of which is covered by a relay station (RS). For example Fig. 1 shows a hexagonal cellular layout with a maximum number of hops, $n_{hops}=3$. Depending on its location in the macrocell, a

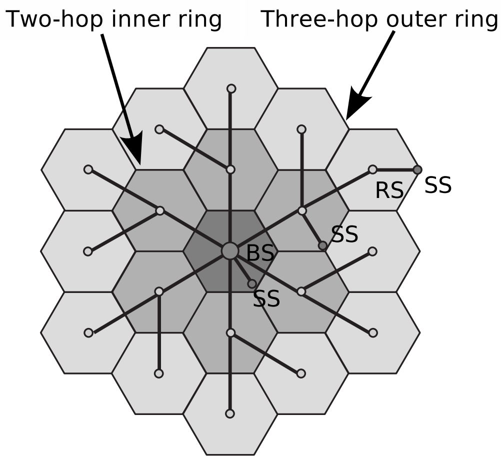

Figure 1: Three-hop hexagonal relay topology.

Table 1: *Model parameters.*

System Parameters	
Carrier frequency	5.8 GHz
Channel bandwidth (W)	10 MHz
Receiver noise figure (F)	6 dB
Receiver noise floor (kTWF)	-98 dBm
Maximum transmit power	30 dBm
Omni antenna gain	9 dBi
Directional antenna gain	17.5 dBi
Directional antenna front-back ratio	25 dB
Other losses (cable)	6 dB
Link margin	6 dB
Duplexing	TDD
Multiple access	TDMA
PHY mode	802.16 OFDM
Macrocell BS antenna height	32 m
Microcell BS antenna height	10 m
Microcell RS antenna height	10 m
SS antenna height	1.5 m
Building height	12 m
Building to building distance	50 m
Block size	200 m

given subscriber station (SS) may be served via one hop by the BS, via two hops by the inner ring of RSs, or via three hops by the outer ring of RSs.

Link budgets for the forward link of all hops for numerous such topologies (hexagonal and Manhattan, two to four hops) have been calculated using the parameters summarized in Table 1 (parameters based on [4], [19], [20]). Noise and interference from other microcells (microcells within the centre macrocell and microcells in four surrounding tiers of macrocells) have been included to calculate signal to interference and noise ratios (SINRs) at the receivers on each link (BS-RS, RS-RS, and RS-SS) in the macrocell. Adaptive modulation and coding (AMC) has been applied to determine throughput corresponding to the SINR on each individual link. Throughput used throughout this work considers the overhead in the medium access control (MAC) and physical (PHY) layers, and was derived from [4], [21] (also see [22] for more detail).

2.2. Path Loss Models

Following recommendations in IEEE 802.16-2004 [23] and the Third Generation Partnership Project (3GPP) [19] the radio frequency (RF) path loss models used in this work are: COST231-Hata model for macrocells (when path length is greater than 1 km), and COST231-Walfish-Ikegami for microcells and macrocells when path lengths are less than 1 km. These models are suitable for frequencies up to 2 GHz. Many applications will use 5 GHz unlicensed spectrum, and so we have also used extensions to these models given by [20]. With the model parameters summarized in Table 1, the resulting line-of-sight (LOS) and non-line-of-sight (NLOS) path loss models used in this work are given in Table 2. Shadowing and small scale fading are not explicitly included in these calculations since their effects are averaged out.

We have used a ”dual-slope” model, in which a NLOS or LOS path loss model is chosen based on the path length. If the path length is less than a distance *breakpoint*, then the LOS model is chosen. Otherwise, the NLOS model is chosen. This breakpoint depends on frequency, geometry (exact positions of the stations), and the propagation environment. In this work, we assume a breakpoint of 250 m, which is reasonable for an urban environment. Inter-microcell and inter-macrocell interference is always further away, and thus is always NLOS.

2.3. Calculation of Throughput Regions - General Formulation

We have adapted the rate matrix approach from [24] for the work presented here. A *link rate matrix*, $\mathbf{R_L}$, is defined as

$$\mathbf{R_L} = \{R_{L,ij}\} : i, j\epsilon[1, N_n], i \neq j \quad (1)$$

where N_n is the number of nodes (BS, RSs and SSs) in a network, and $R_{L,ij} \geq 0$ is the rate (throughput) between transmitter i and receiver j, calculated as described in Subsection 2.1. When $R_{L,ij} > 0$ there exists a usable link between nodes i and j. There exist N_s different transmission schemes $\{\mathcal{S}_k\}$ with corresponding rate matrices

$$\mathbf{R}_k = \{R_{ijk}\} : k\epsilon[1, N_s], i, j\epsilon[1, N_n], i \neq j \quad (2)$$

For each $\mathcal{S}_k$, with node A_i as the *original* data source

$$R_{ijk} = \begin{cases} R & \text{if node } A_j \text{ receives at rate } R, \\ -R & \text{if node } A_j \text{ transmits at rate } R, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Each R is drawn from $\mathbf{R_L}$ for the appropriate inter-node link. First, a rate matrix without spatial reuse (non-SR rate matrix), $\mathbf{R'}_k$, is constructed for a particular network topology. Next a rate matrix with spatial reuse (SR rate matrix), $\mathbf{R}_k$, is calculated according to a *compatibility matrix*, described by [9]

Table 2: *RF path loss models, x is path length in metres.*

Environment	2 GHz Loss (dB)	5.8 GHz Loss (dB)
Urban Macrocell NLOS BS to SS, interferers $x>$1000m	$34.5+35.0\log_{10}(x)$	$42.5+35.0\log_{10}(x)$
Urban Microcell NLOS BS to RS, RS to SS 5000m$>x>$breakpoint	$34.5+38.0\log_{10}(x)$	$42.5+38.0\log_{10}(x)$
Urban Microcell LOS BS to RS, RS to SS 20m$<x<$breakpoint	$30.2+26.0\log_{10}(x)$	$38.2+26.0\log_{10}(x)$

$$\mathbf{M_C}=\{M_{C,ij}\}, i,j\epsilon[1,N_n] \tag{4}$$

This matrix describes which nodes may transmit simultaneously without creating excessive interference for each other. This matrix is topology-dependent, and its elements are

$$M_{C,ij}=\begin{cases}1 & \text{if nodes } A_j,\ A_i \text{ may transmit simultaneously,}\\ 0 & \text{otherwise.}\end{cases} \tag{5}$$

The question now is to find a schedule that i) makes the best use of the link rates, and ii) makes the best use of spatial reuse opportunities. This can be formulated as a linear convex optimization problem. All possible transmission schemes, $\{\mathcal{S}_k\}$, are described by the set of rate matrices, $\{\mathbf{R}_k\}$. We now wish to determine what is the best fraction of time to allocate to each scheme. Let vector $\bar{a}=[a_1a_2...a_{N_s}]$ describe the schedule, with a_k being the fraction of time that scheme $\mathcal{S}_k$ is allocated in one complete schedule cycle. We note that

$$0\le a_k\le 1, \forall k\epsilon[1,N_s] \tag{6}$$

$$\sum_{k=1}^{N_s} a_k\le 1 \tag{7}$$

Once the schedule, $\bar{a}$, is determined, a total rate matrix, $\mathbf{R}$, describing the data flow in the network is calculated as

$$\mathbf{R}=\sum_k a_k\mathbf{R}_k \tag{8}$$

This matrix contains the following elements

$$R_{ij}\begin{cases}<0 & \text{node } A_j \text{ is a net source of data,}\\ >0 & \text{node } A_j \text{ is a net sink of data,}\\ =0 & \text{node } A_j \text{ is acting as a relay only.}\end{cases} \tag{9}$$

As before, the row index, i, indicates the original data source and the column index, j, indicates the active node, so if $\mathbf{R}$ is to be a correct description of the network data flow, we must have

$$R_{ij}\begin{cases}\le 0 & \text{when } i=j,\\ \ge 0 & \text{when } i\ne j.\end{cases} \tag{10}$$

The optimization problem can be formulated a number of ways. We have constraints 6 and 7 and further constraints $R_{ij}=0$ for nodes acting as relays (from 9). At first, it seems sensible to use the network sum throughput, R_{net} as the objective to maximize:

$$R_{net}=\sum_{i,j:R_{ij}>0} R_{ij} \tag{11}$$

Although this results in the highest network spectral efficiency, the resulting schedule will always favour one hop SSs (those SSs in close proximity to the BS) over multi-hop SSs (those SSs closer to the cell edge), which is unfair to SSs, and virtually useless. We can force equal throughput to all SSs by adding another set of constraints

$$R_{i,j}=R_{k,l}: i\ne j, k\ne l \tag{12}$$

when nodes i or j, and k or l are SSs.

In order to find throughput regions, we define a *credit matrix* $\mathbf{C}$ with elements c_{ij} chosen to weight the link between nodes i and j. The product $c_{ij}\cdot R_{ij}$ is used in 11 in place of R_{ij}. Appropriate choice of credit weights gives more distant multi-hop nodes a better chance of receiving service. Adjusting the credit matrix allows the tracing of the throughput regions and scheduling for non-uniform traffic distribution. One method of weighting adapts the idea of transport capacity from [25], in which credit is given for the distance a bit travels.

$$c_{ij}=\frac{d_{ij}}{r}: i,j\epsilon[1,N_n] \tag{13}$$

where d_{ij} is the distance between nodes i and j.

3. Calculation of Throughput Regions - Three Hop Hexagonal Tree Example

We have performed calculations for two, three and four hop topologies, using both hexagonal and Manhattan layouts. The example presented here uses the three hop network shown in Fig. 1 and considers the forward links from the BS to SSs. We lump SSs into one SS per microcell and calculate a sum throughput per microcell. With the symmetric topology in the figure, all BS-SS and RS-SS throughputs are equal, all BS-RS throughputs are equal, and all RS-RS throughputs are equal. Using the parameters in Table 1 with a macrocell of 1 km circumscribed radius (see [22]), an example set of throughputs are $R_{BS-SS} = R_{RS-SS} = 26.3$ Mb/s, $R_{BS-RS} = 11.9$ Mb/s, and $R_{RS-RS} = 11.9$ Mb/s. With symmetry and the BS as the single data source, the general formulation described above simplifies greatly. This results in a network with one data source and 38 data sinks, so the throughput region has 38 dimensions.

In order to plot the tradeoff between serving one-hop, two-hop and three-hop regions, we sum up the total throughputs in each of the two-hop (inner) and three-hop (outer) rings, and show some throughput region cross-sections.

Fig. 2(a) shows a two dimensional slice of the one-hop SS to two-hop (inner-ring) throughput region, and the network sum throughput with no spatial reuse. At the point where maximum network throughput is achieved, no throughput occurs to the outer SSs, and although high in spectral efficiency, this network covers only 1/19 of the macrocell area. It is necessary to trade network throughput for coverage. Fig. 2(b) shows the two-hop (inner-ring) throughput vs three-hop (outer-ring) throughput region, and network sum throughput. It shows a similar tradeoff.

Fig. 3 shows the effects of spatial reuse. As three-hop SSs are served, there are opportunities for more than one node to transmit simultaneously.

Calculations for this macrocell without multihop relaying indicate that the best throughput for a SS at the cell edge is 2.90 Mb/s. From the figures, it appears that although network throughput is sacrificed greatly for the purpose of serving the farthest SS, we find that the network throughput when serving the farthest SSs has more than doubled to 5.95 Mb/s with the use of multi-hop relaying.

4. Conclusions

This paper has presented a method of analysis of realistically achievable network throughput (with consideration of overhead in PHY and MAC layers) of multi-hop relaying cellular networks. Throughput regions showing the tradeoff in serving subscriber stations near to and far from the base station have been calculated. A convex optimization problem is solved to find the best schedule of transmission schemes to maximize the network throughput subject to different constraints. We introduce constraints to force the network to be fair, based on the distribution of offered user traffic in three regions: the one-hop region within the coverage area of the BS, the two-hop region covered by the inner ring of RSs, and the three-hop region covered by the outer ring of RSs.

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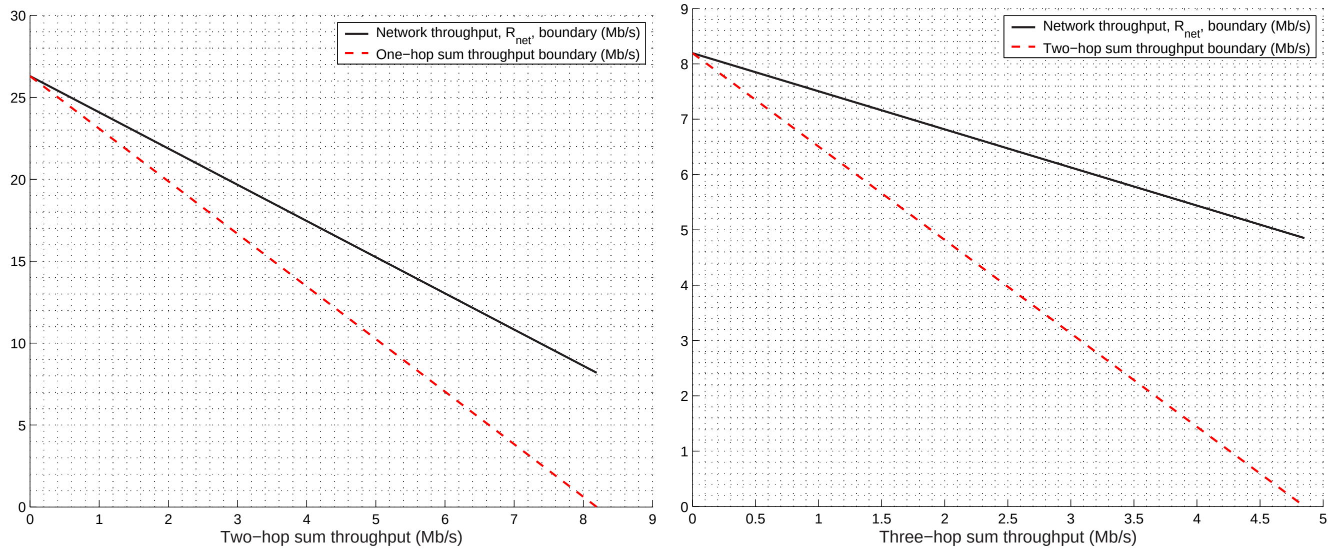
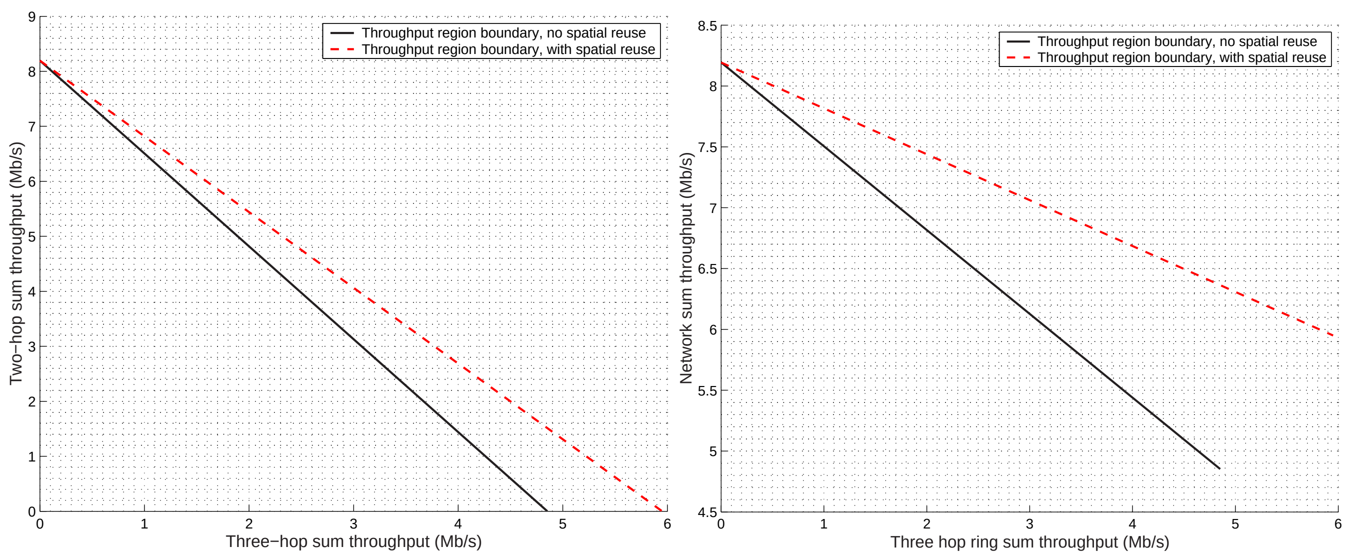


(a) One-hop nodes vs inner ring (two-hop) nodes.

(b) Inner ring (two-hop) nodes vs outer ring (three-hop) nodes.

Figure 2: Throughput region boundaries - no spatial reuse.



(a) Inner ring (two-hop) nodes vs outer ring (three-hop) nodes.

(b) Network throughput, R_{net}.

Figure 3: Throughput region boundaries - with and without spatial reuse.

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